

QED and QCD

Tutorial Questions

Sheet 2

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Note: if pressed for time ignore the “arbitrary gauge” part in the first question. The second question is partly a repetition of what was already presented in the lectures.

Tutorial Questions 1: Bhabha scattering

Draw all the tree-level diagrams for Bhabha-scattering

$$e^+(p_1)e^-(p_2) \rightarrow e^+(p_3)e^-(p_4), \quad (1)$$

and give the expression for the scattering amplitude in Feynman gauge. What happens in an arbitrary gauge?

Tutorial Questions 2: Muon production

Let us consider

$$e^+(p_1)e^-(p_2) \rightarrow \mu^+(p_3)\mu^-(p_4), \quad (2)$$

in the limit $m_e = 0$.

a) Show that the matrix element squared is

$$\sum |\mathcal{M}|^2 = \frac{e^4}{(p_1 + p_2)^4} \text{tr} \left[\gamma \cdot p_1 \gamma^\mu \gamma \cdot p_2 \gamma^\nu \right] \text{tr} \left[(\gamma \cdot p_4 + M) \gamma_\mu (\gamma \cdot p_3 - M) \gamma_\nu \right], \quad (3)$$

when summed over spins.

b) Show that

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \sqrt{1 - \frac{4M^2}{s}} \frac{1}{4} \sum |\mathcal{M}|^2, \quad (4)$$

with $s = (p_1 + p_2)^2$.

c) If you have time, evaluate the traces to show

$$\sum |\mathcal{M}|^2 = \frac{2e^4}{s^2} \left[(p_4 \cdot p_2)^2 + (p_4 \cdot p_1)^2 + M^2(p_1 \cdot p_2) \right]. \quad (5)$$

d) Show that, in the centre-of-mass frame with scattering angle θ ,

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{64\pi^2 s} \sqrt{1 - \frac{4M^2}{s}} \left[1 + \left(1 - \frac{4M^2}{s} \right) \cos^2 \theta + \frac{4M^2}{s} \right] \quad (6)$$

e) Find an expression for the total cross section in the high-energy limit where the mass of the muon can be neglected.