UNIVERSITY OF LIVERPOOL

QED and QCD
Tutorial Questions
Sheet 2

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Note: if pressed for time ignore the "arbitrary gauge" part in the first question. The second question is partly a repetition of what was already presented in the lectures. You can focus on a and b

Tutorial Questions 1: Bhabha scattering

Draw all the tree-level diagrams for Bhabha-scattering

$$e^+(p_1)e^-(p_2) \to e^+(p_3)e^-(p_4)\,, \tag{1}$$

and give the expression for the scattering amplitude in Feynman gauge. What happens in an arbitrary gauge?

Tutorial Questions 2: Muon production

Let us consider

$$e^+(p_1)e^-(p_2) \to \mu^+(p_3)\mu^-(p_4)\,, \tag{2}$$

in the limit $m_e = 0$.

a) Show that the matrix element squared is

$$\sum |\mathcal{M}|^2 = \frac{e^4}{(p_1+p_2)^4}\mathrm{tr}\Big[\gamma\cdot p_1\,\gamma^\mu\,\gamma\cdot p_2\,\gamma^\nu\Big]\mathrm{tr}\Big[(\gamma\cdot p_4 + M)\,\gamma_\mu\,(\gamma\cdot p_3 - M)\,\gamma_\nu\Big]\,, \tag{3}$$

when summed over spins.

b) Show that

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 s}\sqrt{1-\frac{4M^2}{s}}\frac{1}{4}\sum|\mathcal{M}|^2\,, \tag{4}$$

with $s = (p_1+p_2)^2$.

c) If you have time, evaluate the traces to show

$$\sum|\mathcal{M}|^2 = \frac{2e^4}{s^2}\Big[(p_4\cdot p_2)^2 + (p_4\cdot p_1)^2 + M^2(p_1\cdot p_2)\Big]\,. \tag{5}$$

d) Show that, in the centre-of-mass frame with scattering angle θ,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{e^4}{64\pi^2 s}\sqrt{1-\frac{4M^2}{s}}\left[1+\left(1-\frac{4M^2}{s}\right)\cos^2\theta + \frac{4M^2}{s}\right] \tag{6}$$

e) Find an expression for the total cross section in the high-energy limit where the mass of the muon can be neglected.