

QED and QCD

Tutorial Questions

Sheet 3

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Tutorial Questions 1: Compton scattering Consider Compton scattering

$$e(p_1)\gamma(p_2) \rightarrow e(p_3)\gamma(p_4). \quad (1)$$

- a) Derive the amplitude for Compton scattering and verify that it is gauge-invariant.
- b) Using the matrix element squared from the lecture

$$\sum |\mathcal{M}|^2 = 8e^4 \left(\frac{p_1 \cdot p_2}{p_1 \cdot p_4} + \frac{p_1 \cdot p_4}{p_1 \cdot p_2} + 2m^2 \left(\frac{1}{p_1 \cdot p_2} - \frac{1}{p_1 \cdot p_4} \right) + m^4 \left(\frac{1}{p_1 \cdot p_2} - \frac{1}{p_1 \cdot p_4} \right)^2 \right). \quad (2)$$

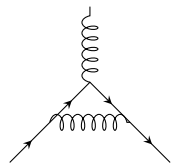
Working in the centre-of-mass system, in the limit where the electron mass m can be neglected, show that the matrix element squared is dominated by backward scattering, $\theta \simeq \pi$, where θ is the scattering angle of the photon.

- c) Use the matrix-element squared for Compton scattering to obtain the matrix-element squared for the annihilation process $ee \rightarrow \gamma\gamma$. Again work in the centre-of-mass frame and show that, in the high-energy limit $E \gg m$

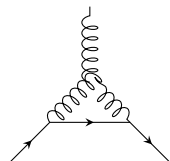
$$\sum |\mathcal{M}|^2 = 16e^4 \frac{1 + \cos^2 \theta}{\sin^2 \theta}. \quad (3)$$

Tutorial Questions 2: Colour factors

Show that the colour factors in the following diagrams are



$$= t^c t^a t^b \delta^{bc} = -\frac{1}{2N_c} t^a, \quad (4)$$



$$= i f^{abc} t^b t^c = -\frac{1}{2} C_A t^a. \quad (5)$$

Tutorial Questions 3: Quark scattering

Calculate the spin-summed matrix element squared for quark scattering $ud \rightarrow ud$ in the limit of vanishing quark masses.